Summation in SPF Calculations of the SPF-290AS™

With the identification of a satisfactory substrate for in-vitro measurement of sunscreen protection factors (SPF), manufacturers of spectroradiometers dedicated to SPF measurements have used the equations described in A new substrate to measure sunscreen protection factors throughout the ultraviolet spectrum, B.L Diffey and J. Robson, J. Soc. Cosmet. Chem., 40, 127-133 (May/June 1989) as the basis for measurement calculations. Over the years, spectroradiometric software has provided other useful data to aid the photobiologist in their quest to develop new sunscreen products.

Recently, customers have expressed concern that companies have been cavalier in their use of simple summation to approximate integrals but use integral symbology in documentation and reporting.

The "integral" can be defined as designating simply a continuous sum. When this is based on an analytic equation then, in principle, an 'exact' computation is possible mathematically. However, when it is based upon a discreet set of points, such as data measurements, then it is by definition a sum, although it approaches an integral, and integral notation is often used to designate it. This sometimes causes confusion. Somewhere in between is the case of tabulated data included in the expression, such as solar irradiance, etc. There is no analytic expression, but there may be values available at extremely small intervals. A choice is made as to how many points it makes sense to include in the computation, but again this is a sum. By using techniques such as trapezoidal approximations or triangular averaging at the end points, what is accomplished is that the sum is a slightly better approximation to an 'ideal integral'. However, it is understood that the ideal integral cannot actually be carried out, but better approximations to it can be achieved. How good those need to be requires that judicious choices be made, since truly continuous data is not available.

Solar Light has taken care to define its use of notation: summation (Σ) and integration (∫) here signify a slightly different computation. When Σ appears in an equation the software implementation is exactly as described by the equation.

When we use an integral symbol we are using a software algorithm to better approximate the integral. However, for the reasons described above, sums are used in both methods and the notation distinguishes them.

For example, the equation we use for UVA/UVB ratio is:

$$UVA/UVB = \frac{\int_{320}^{400} Log_{10} MPF_{\lambda} d\lambda / \int_{320}^{400} d\lambda}{\int_{290}^{320} Log_{10} MPF_{\lambda} d\lambda / \int_{290}^{320} d\lambda}$$

Equation 1

To calculate the numerator we use the rectangular technique to approximate the integral as follows:

$$UVA = \frac{\left[\sum_{325}^{395} Log_{10}(MPF_{\lambda}) + .5[Log_{10}(MPF_{320}) + Log_{10}(MPF_{400})]\right] * d\lambda}{80}$$

Equation 2

SPF equipment manufacturers are also being encouraged to be more accurate in their modeling, mainly because users are becoming more sophisticated in their use of data gained from in-vitro testing. Thus, Optometrics offers the choice of using the traditional equation for SPF as published by Diffey or an integral approximation. Therefore, beginning with WinSPF™ V 2.1, the user can select from two methods represented by the following equations:

$$SPF_{Scan} = \frac{\sum_{290}^{400} E_{\lambda} B_{\lambda}}{\sum_{290}^{400} \frac{E_{\lambda} B_{\lambda}}{MPF_{\lambda}}}$$

Equation 3

or

$$SPF_{Scan} = \frac{\int_{290}^{400} E_\lambda B_\lambda d\lambda}{\int_{290}^{400} \left[\frac{E_\lambda B_\lambda}{MPF_\lambda} \right] d\lambda}$$

Equation 4
implemented as follows:

$$SPF_{Scan} = \frac{\left[\sum_{295}^{395} E_\lambda B_\lambda + 0.5\{E_{290}B_{290} + E_{400}B_{400}\} \right] d\lambda}{\left[\sum_{295}^{395} \frac{E_\lambda B_\lambda}{MPF_\lambda} + 0.5\left\{ \frac{E_{290}B_{290}}{MPF_{290}} + \frac{E_{400}B_{400}}{MPF_{400}} \right\} \right] d\lambda}$$

Equation 5

In some cases, the differences in the results between the two methods is insignificant while in others, the user may find the use of equation 5 to be clearly preferable to equation 3. Table 1 shows samples with wide-ranging protection factors and the affects of using the two methods of calculating SPF.

Sample	SPF (Summation)	SPF (Rect. Approx.)	% Difference
A	4.71	4.72	0.2
B	8.61	8.66	0.6
C	12.50	12.64	1.1
D	25.78	26.31	2.1
E	49.19	51.54	4.2
F	161.77	161.85	0.05

Note that samples A through E are for lotion and cream sunscreen products with a typical spectral distribution of higher UVB absorption than UVA absorption, i.e. UVA/UVB from 0.25 – 0.50. Sample F is a fabric with a UVA/UVB ratio of 0.94.

To be consistent, the erythemal UVA protection factor included in WinSPF™ reports is also available as a straight summation or rectangular approximation of the integral.

Using WinSPF™ from Optometrics, the selection between the methods can be made in the Sample Setup form or the Change Calc. Method form by selecting either Rectangular Approximation or Rough Sum. To avoid arbitrary changes when reviewing sample data, Rough Sum is set as the default. If one wishes the older data to be recalculated using rectangular approximation, use Change Calc. Method and then save the data, overwriting the previous file.